MAP555 - Signal processing- Exercise set 1
Fourier Transform and Analog filtering

Exercise 1 Fourier Transform and frequency representation

Use the FT properties to compute the Fourier transform of the following signals and plot their modulus:

1. $x(t) = 2\sin(2\pi f_0 t)$
2. $x(t) = \sin(4\pi f_0 t + \pi/3)$
3. $x(t) = 2\cos(2\pi f_0 t) + \sin(3\pi f_0 t)$
4. $x(t) = 1$

Exercise 2 Fourier Transform computation

For all the following signals, plot the signal, compute their Fourier Transform and plot its amplitude. Avoid computing the integral when possible.

1. $x(t) = e^{-at}\Gamma(t)$ with $a > 0$
2. $x(t) = e^{-a|t|}$ with $a > 0$
3. $x(t) = \frac{1}{1+t^2}$
4. $x(t) = \frac{1}{2-2t+t^2}$
5. $x(t) = \frac{t}{(1+t^2)^2}$

Exercise 3 Mean filter

The mean filtering parametrized by T computes the average value of the signal on a sliding window of size T. As such its impulse response can be expressed as

$$h(t) = \Pi_T(t - T/2) = \begin{cases} \frac{1}{T} & \text{for } 0 \leq t \leq T \\ 0 & \text{else} \end{cases}$$

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1. Plot the impulse response, is the system causal?
2. Compute the output of the system for the following inputs:
   i. $x(t) = 1$
   ii. $x(t) = \Gamma(t)$ (unit step function).
   iii. $x(t) = cos(2\pi f_0 t)$
3. Express the output of the system for $x(t) = cos(2\pi f_0 t)$ as $y(t) = A(f_0)cos(2\pi f_0 t + \phi(f_0))$.
4. Plot $|A(f_0)|$ as a function of f_0 for $T = 1$. Also plot $\phi(f_0)$.
5. Compute the Frequency response of the system $H(f)$, what is its relation with $A(f)$) and $\phi(f)$?

Exercise 4 Frequency response of electrical systems

Compute the Frequency response of the following electrical systems and plot their Bode plot. Interpret each system and discuss the type of filter implemented (low-pass, high-pass, ...) .

Complex impedance:

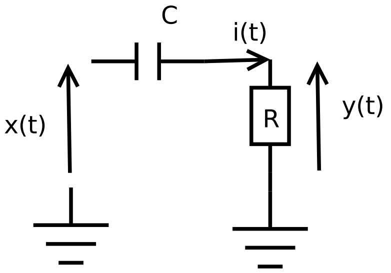
Resistor Real impedance $Z_R = R$.

Capacitor $Z_C = \frac{1}{jCw} = \frac{1}{jC2\pi f}$

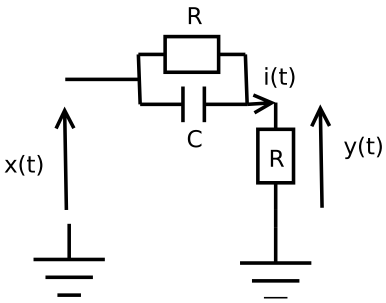
Inductance $Z_L = jLw = jl2\pi f$

Impendance in series $Z_{eq} = Z_1 + Z_2$

Impendance in parallel $Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$



(1)



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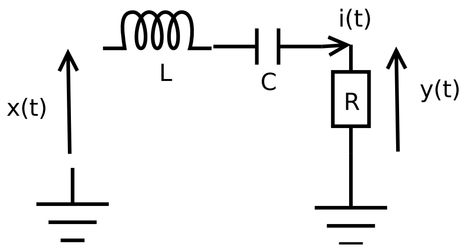


(3) $w_n = \frac{1}{\sqrt{LC}}, z = 1$ for plot.

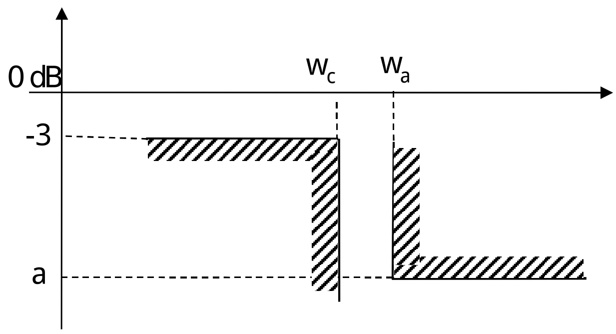
Exercise 5 Filter design

1. We recall that the squared modulus of the frequency response of a Butterworth filter of order N can be expressed as :

$$|\tilde{H}(w)|^2 = \frac{1}{1 + \left(\frac{w}{w_c}\right)^{2N}}$$

Plot the magnitude part of the Bode diagram of the filters for different values of N.

2. We want to design a filter that respects the following constraints:

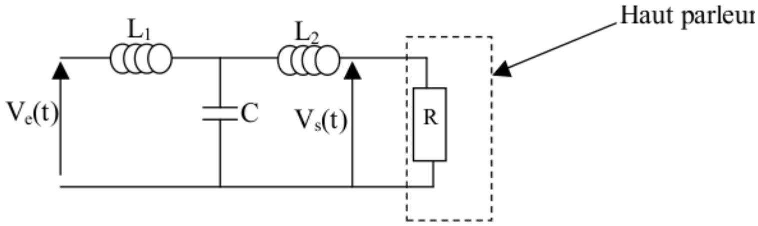


Using the expression of the Frequency response above, compute the minimal order of the filter that respects the constraints (as a function of the parameters w_c, w_a et a)

Exercise 6 Filter realization on audio speakers

In order to cover the large bandwith of the human hear (and sell expensive devices), audio loudspeakers usually have several drivers. Each driver have a different size and will cover one part of the bandwith of the full speaker (woofer for low frequency, mid-range driver for medium frequencies and tweeter for high frequencies). each driver is preceded by a filter and we will study the filters for the woofer and tweeter drivers in this exercise.

1. Study of the Woofer
   - Electronic system:

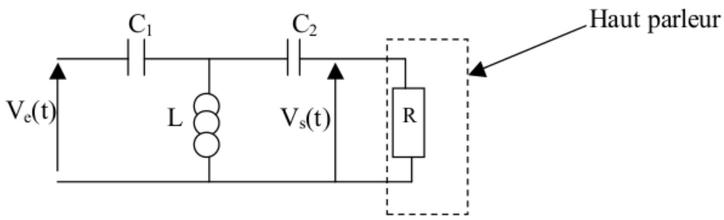


   - Express the frequency response $H_1(w)$ as a function of the system parameters.
   - A Butterworth or order 3 is used in order to avoid ripples in the bandpass. Its normalized frequency response can be expressed as

$$H(w) = \frac{1}{\left(\left(\frac{jw}{w_0}\right) + 1\right)\left(\left(\frac{jw}{w_0}\right)^2 + \frac{jw}{w_0} + 1\right)}$$

     Express the unormalized frequency response with cutoff frequency w_0 and deduce the parameters L_1, L_2 and C as a function of R and w_0.
   - Compute the numerical values for a normalized resistor 8 Ohm for the driver and a cutoff radial frequency of $w_0 = 6000$ rad/s.
   - Draw the Bode plot of the system.

2. Study of the Tweeter
   - Electronic system:



   - Express the frequency response $H_1(w)$ as a function of the system parameters.
   - We now still use a Butterworth filter but we need to transform it to its hight pass version. Express the normalized frequency response for a high pass order 3 Butterworth filter using the transformation $jw \to 1/jw$. Deduce the parameters C_1, C_2 and L as a function of R and w_0.
   - Compute the numerical values for a normalized resistor 8 Ohm for the driver and a cutoff radial frequency of $w_0 = 12000$ rad/s.
   - Draw the Bode plot of the system.